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Geology, Geometry, and Effective Flow

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Abstract

The effects of small-scale structure are frequently ignored in reservoir simulation, although they may have a significant effect on hydrocarbon recovery. Many sandstones exhibit lamination, and in such rock structures, permeability may vary by an order of magnitude over distances of a centimetre or less. Frequently, the laminae are inclined with respect to the pressure gradient, which gives rise to crossflow within the unit. In this case a tensor may be used to represent the effective permeability of the bed.

In this paper the effect of small-scale geological structure on single phase flow is investigated using a model of a crossbedded unit to determine which factors have the greatest effect on crossflow. In addition, a range of other types of bedding are considered, including models with stochastic variation. We have found that, in general, the geometry of the sedimentary structure has a significant effect on flow. The crossflow is greater (and therefore tensors are more likely to be necessary) when the angle of the laminae is large relative to the pressure gradient (up to 45^o), when the permeability contrast between laminae is large, or when the structure is asymmetric.

1. Introduction

Small-scale sedimentary structures, such as laminae and beds have been described in detail by several authors[1,2]. However, it was not usually appreciated that these laminated structures can have a significant effect on fluid flow. Studies using probe permeameters[3,4] have shown that permeabilities may vary by an order of magnitude or more between laminae. On the other hand, reservoir engineers are often unfamiliar with detailed rock structures and do not include these small-scale features in their flow models. Also, it is considered too time-consuming to build models starting at scales of mm or cm, and to scale up the flow properties (such as relative permeability and capillary pressure) to the full field scale. Instead, reservoir models commonly focus on the stochastic generation of geological units, at scales of metres or larger, with the implicit assumption that rocks can be modelled as homogeneous at smaller scales. However, small-scale features, such as laminae, can have a significant effect on hydrocarbon recovery due to capillary trapping[5,6]. If the processes of fluid flow are to be accurately modelled and understood, it is important for geologists and engineers to work together to create models which are geologically realistic at all scales.

In this paper, we derive permeability models for a variety of sedimentary structures, (see for example Figure 1), starting at the lamina scale with gridblocks of mm-cm size. We show how different structures affect single phase flow, and how these effects may be captured by using tensor effective permeabilities, to simulate flow at the bedform scale (dm-m).

2. Effective Permeability

Effective permeability, k_{eff}, is defined as the permeability of a homogeneous block which, under

References and illustrations at end of paper.

the same pressure boundary conditions, will give the same average flows as the region the block is replacing. It is assumed that the flow is uniform (linear, steady-state flow), because effective permeability may not be uniquely defined for non-uniform flow[7]. The effective permeability is not simply an intrinsic property of a rock but also depends on the boundary conditions (i.e. the permeabilities and pressure distributions within the surrounding rock). In general, a flow simulation with appropriate boundary conditions is required in order to calculate the effective permeability. However, there are some well known cases where the effective permeability may be calculated analytically. For uniform flow along continuous parallel layers, the effective permeability is the arithmetic average, k_a, given by:

$$k_a = \frac{\sum_{i=1}^{n} k_i t_i}{\sum_{i=1}^{n} t_i},$$

where i=1,2, ...n is the number of layers, and k_i and t_i are the permeability and thickness, respectively, of the i-th layer[8]. For uniform flow perpendicular to continuous parallel layers, the effective permeability is given by the harmonic average, k_h, where:

$$k_h = \left[\frac{\sum_{i=1}^{n} t_i/k_i}{\sum_{i=1}^{n} t_i} \right]^{-1}.$$

In the case of correlated random permeabilities with a log-normal permeability distribution, the effective permeability is approximated by the geometric mean, k_g, in 2D[9], where

$$k_g = \exp\left[\sum_{i=1}^{n} \ln(k_i)/n \right].$$

In 3D, k_{eff} also depends on the variance, σ^2, of ln(k), and is approximated by:

$$k_{eff} = k_g(1 + \sigma^2/6).$$

The above formulae for k_{eff} are very useful, but they do not generally apply due to the more complex geometry and variability which occurs in nature. Very few rocks have permeability distributions which are exactly ordered, or purely random. Most show a combination of lamination with stochastic variation. Furthermore, the flow direction is commonly at an angle to the laminae, and this can give rise to crossflow (Figure 2). The effective permeability may then be represented by a tensor, $\underline{\underline{k}}$, to take account of crossflow.

$$\underline{\underline{k}} = \begin{bmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{bmatrix} \text{in 2D,}$$

or

$$\underline{\underline{k}} = \begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{bmatrix} \text{in 3D.}$$

For example, k_{xy} measures the amount of flow in the x-direction due to a pressure gradient in the y-direction, and so on.

For flow at an angle to continuous parallel layers, the effective permeability tensor may still be calculated analytically, by rotation of the axes. (Co-ordinate transformations are treated in many mathematical textbooks[10].)

$$\underline{\underline{k}} = \begin{bmatrix} k_a\cos^2\theta + k_h\sin^2\theta & (k_a-k_h)\sin\theta\cos\theta \\ (k_a-k_h)\sin\theta\cos\theta & k_a\sin^2\theta + k_h\cos^2\theta \end{bmatrix},$$

where k_a and k_h are the arithmetic and harmonic averages respectively, and θ is the angle of the layers to the x-axis, as shown in Figure 2. Here we are assuming that the area considered is far from any boundaries. We can see from this formula that the off-diagonal terms, which reflect crossflow, will become larger as

a) θ increases from 0^o to 45^o,

and

b) (k_a-k_h) increases, or the permeability contrast between lamina increases.

Figures 3 a) and 3 b) show the effect of varying the lamina angle and the permeability contrast respectively.

If significant crossflow is present, it may need to be included in simulations at a larger scale. ("Significant" crossflows are considered to be those greater than 10% of the direct flow). This requires calculation of flow using tensor permeabilities. Such simulations require a 9-point finite difference scheme instead of the 5-point one in 2D, and in 3D require a 19-point scheme instead of a 7-point one. (The equations for simulating flow using tensors in a 2D model are presented in Reference 11.) A simulation using tensors is, consequently, more time-consuming than one which only uses single permeability values (scalars) or one which uses a permeability in each direction (vector). Therefore, it is important to assess in advance, which structures require full tensors to accurately describe their flow properties and which do not. In the following sections we show which kinds of bedding are most likely to require tensors and which are not.

3. Crossflow in Crossbedded Structures

To illustrate the effect of bed geometry on crossflow, we have chosen a "crossbed model" (i.e. a section of a cross-bedded sandstone). This model is derived from a study of the Ardross outcrop, near St. Monance, Fife, Scotland. The crossbeds were deposited under deltaic conditions, and comprise alternating layers of high and low permeability. Probe permeameter measurements of a sample of this rock (Figure 4) indicate that the permeability of the micaceous laminae is approximately 100 mD, and that of the sandy laminae is about 1200 mD. We have assumed that the permeability within the laminae is homogeneous and isotropic[12]. The shape of the laminae was digitised from a photograph (Figure 1). The average size of a crossbed is 1.0 m x 0.8 m, and this is modelled using a grid of 240 x 220 blocks. Because of the complex shape of the bedform, the tensors for these crossbeds cannot be calculated analytically. We have used a periodic boundary condition method, similar to that described by Durlofsky[13], but with a finite difference method for setting up the pressure equations[14]. The base case model (model 1) is shown in Figure 5. The flow through this model is illustrated using flow vectors, the length of the arrow being proportional to the volumetric flow rate. The tensor for this model, along with those for the other models, is given in Table 1. The ratio k_{xy}/k_{xx} equals 0.21, indicating that there is significant crossflow in this model.

Four modifications to the original model were made to test the sensitivity of the tensor to permeability distribution (Table 1). Model 2 (Figure 6) has a high permeability bottomset of 1200 mD, instead of a low permeability value of 100mD. The overall permeability of this model is higher, but the amount of crossflow, as measured by k_{xy}/k_{xx}, is similar. In model 3 (Figure 7), the permeability of the bottomset has been reduced from 100 mD to 1 mD. This inhibits crossflow, and so the k_{xy}, k_{yx} and k_{yy} terms become negligibly small. Model 4 contains tangentially graded foresets, where the permeability increases from 100 mD at the bottom of the set to 1200 mD at the top (Figure 8). In this case, the crossflow is only a few percent, because most of the flow is channelled along the higher permeabilities of the upper part of the model. Finally, in model 5 which is similar to model 1, the width of the low permeability laminae were reduced by half. The crossflow in this model is less than that in models 1 and 2, but is still significant. Thus, we conclude that crossflow in crossbeds is generally significant (>20%), but may be less important where there are very low permeability bottomsets, or where there is tangential grading, giving rise to a high permeability region at the top of the crossbed unit.

It is common practice to scale up permeability using a combination of arithmetic, geometric and harmonic averages[15]. For example, the arithmetic average of a permeability sample may be used to calculate the effective permeability for bed-parallel flow, and the harmonic average may be used for flow transverse to the bedding. This procedure is correct for a layered model, as shown in section 2. To test the validity of this approach in crossbedded systems, we calculated k_a, k_g and k_h for each of the models. The results are presented in Table 2. In all cases, the arithmetic average overestimated the k_{xx} tensor value. This effect is most severe in the case of model 3 (very low permeability bottomset) with $k_a/k_{xx} = 1.45$, but the effect is only slight for model 4 (tangential grading) where k_a is only 3% larger than k_{xx}. Comparing k_h with k_{yy}, we find that, with the exception of model 3, k_h underestimates k_{yy}. This effect is slight, however, in the case of model 4. We conclude therefore that only in the case of model 4 can the effective permeability be represented adequately (to within a few percent) using k_a for flow in the horizontal direction and k_h for flow in the vertical direction. The errors in using k_a and k_h instead of k_{xx} and k_{yy} are greater than 10% in the other cases, and are sometimes significantly larger. It can be seen from Table 2 that k_g, which is a good approximation for the effective permeability of a random distribution, was inappropriate for the crossbedded models tested here.

4. Crossflow in Other Sedimentary Structures

A variety of other types of sedimentary structures have been modelled to assess whether or not crossflow is important, namely, hummocky cross-stratification, ripple lamination with stochastic variability, and a 3D crossbed model.

a) Hummocky Cross-stratification

Hummocky cross-stratification is common in shallow marine environments[16]. The bedform is approximately symmetrical (see model in Figure 9). Hence, any crossflow generated in the first half of the model is balanced by crossflow in the opposite direction in the second half of the model. The result is that there is no net crossflow, so tensors are not required for this type of model (provided the size of the gridblocks equals the size of the bedform or multiple bedforms).

b) Ripple Lamination with Stochastic Permeability

As perfectly regular lamination does do not occur in nature, we have tested models which combine regularly deterministic laminae with a stochastic permeability field. The first of these models is a stochastic ripple model, shown in Figure 10. The model is constructed by repeating a ripple template to create a field in which the ripple properties can vary in space. 10 x 10 ripple template blocks, each of 3.2 cm x 2.0 cm, were modelled. In this case, the high permeability component of the template was kept constant throughout the model, at 100 mD but the low permeability component was varied, using a correlated random function, to produce a model which looked similar to images of the ripples in the Ardross Outcrop. (It is assumed that the dark-coloured laminations have lower permeability.) The net crossflow for this model is significant (k_{xy}/k_{xx}=0.12) and is equal to the amount of crossflow for a deterministic model with constant permeability contrast.

c) Ripple Lamination with Stochastic Structure

In these models, we superimposed different amounts of stochastic variation on the deterministic ripple model, so that the laminae were no longer continuous. As anticipated, the amount of crossflow decreases as the amount of stochastic variation (measured by the standard deviation, σ) increases. When σ = 0.5 (Figure 11a), k_{xy}/k_{xx} = 0.15, but when σ = 2.0 (Figure 11b), k_{xy}/k_{xx} = 0.10 (Table 3). We found that the amount of crossflow remains significant while the standard deviation of the deterministic component is greater than that of the stochastic component[14]. In practice, we noticed that if the laminae are easily identifiable they are likely to generate significant crossflow. In the models where the stochastic variation dominates, we might expect that the geometric average of the permeabilities would be a good estimate of the effective permeability. However, we found that this was not the case. Table 3 shows that, as the stochastic component was increased, the k_{xx} and k_{yy} values remained approximately constant, and did not converge on k_g.

d) 3D Crossbedded Model

Finally, in order to evaluate the effects of a third dimension, a 3D crossbedded model was tested (Figure 12). This model consists of alternating layers of 20 mD and 100 mD. The model is symmetrical in the x-y and x-z planes, so the k_{xy}, k_{yx}, k_{xz}, k_{zx} terms are zero, as shown in Figure 12. The ratio k_{yz}/k_{yy} is less than 10%, so the amount of crossflow in the yz plane is quite small. However, this amount of crossflow is likely to be an underestimate, because the 3D grid used was quite coarse, (due to memory limitations in the computer). In future, we plan more detailed calculation of 3D models to test the influence of 3D effects on crossflow.

5. Conclusions

We have found that the effective permeability is strongly influenced by the geometry of the bedding, and that conventional averaging techniques may give misleading estimates. Tests on crossbedded models showed that:

- k_a and k_h are usually poor approximations for k_{xx} and k_{yy} (in crossbedded sandstones);
- k_g is also a poor estimate of effective permeability, even when stochastic variation is superimposed on the structure.

A tensor effective permeability is often required to capture crossflow. Tensors are likely to be necessary when:

- laminae are inclined at a large angle (up to 45°) to the pressure gradient,
- there is a high permeability contrast,
- the laminae are of approximately equal thickness.

Tensors are less important when:

- the bedform is symmetrical,
- there are horizontal barriers (e.g. low permeability bottomsets),
- there are high permeability streaks or pathways (tangentially graded crossbeds).

6. Discussion

In this study, we have concentrated on the influence of primary depositional features on fluid flow. We have not included the effects of faults, fractures and diagenetic changes. However, tensors may also be used to take account of crossflow which may arise due to any such features, provided the spatial permeability structure can be quantified. We infer that tensors are most likely to be necessary when structure (bedding or faults) are inclined at a high angle (up to 45°) to the pressure gradient.

Once tensor effective permeabilities have been determined for a structural permeability field, they should be incorporated into simulations at larger scales. Because sediment bedforms are frequently repeated over scales of 10s of meters or more, the same form of tensor may be used to represent the effective permeability over a whole formation. The tensor may, however, require to be scaled to allow for permeability trends, or it may be varied stochastically to take account of natural variation in the rock. The number of scale-up stages required depends on the geology of the reservoir. At some scales, the net crossflow may be negligible, and a diagonal tensor is produced. It still may be important, however, to include the crossflow terms from smaller scales in the calculations so that this diagonal tensor is determined correctly.

If a rock structure has a significant effect on single phase flow, the effect on multi-phase flow is likely to be even more important. If we consider oil and water, which are immiscible fluids, the flow through the medium will also be affected by gravity and capillary forces. The amount of crossflow will therefore be different for the oil and the water phases, and a separate tensor will be required to describe each phase. Thus to make accurate prediction of oil recovery in crossbedded sandstones, two-phase tensor simulations may be required[17]. It is clear that reservoir simulation requires the combined efforts of both geologists and engineers, so that flow can be modelled accurately, at all scales.

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Table 1

Model Number	Model Description	k_{xx}	k_{yy}	k_{xy}	k_{xy}/k_{xx}
1	base case	534	232	114	0.213
2	high perm. bottomset	685	306	148	0.216
3	very low perm. bottomset	463	9	4	0.009
4	tangential grading	823	404	21	0.026
5	thin low perm. laminae	765	337	133	0.174

Table 2

Model Number	Model Description	k_a	k_g	k_h	$k_{eff(NF)}$
1	base case	677	368	193	509
2	high perm. bottomset	777	461	229	622
3	very low perm. bottomset	668	242	11	480
4	tangential grading	848	647	395	823
5	thin low perm. laminae	892	599	295	738

Table 3

Std. Dev.	k_{xx}	k_{yy}	k_{xy}	k_g
0.0	47.5	33.5	7.1	50.2
0.5	49.5	35.7	7.2	53.0
1.0	48.3	35.9	6.5	53.5
1.5	46.9	36.1	5.5	54.2
2.0	45.7	36.3	4.4	55.0

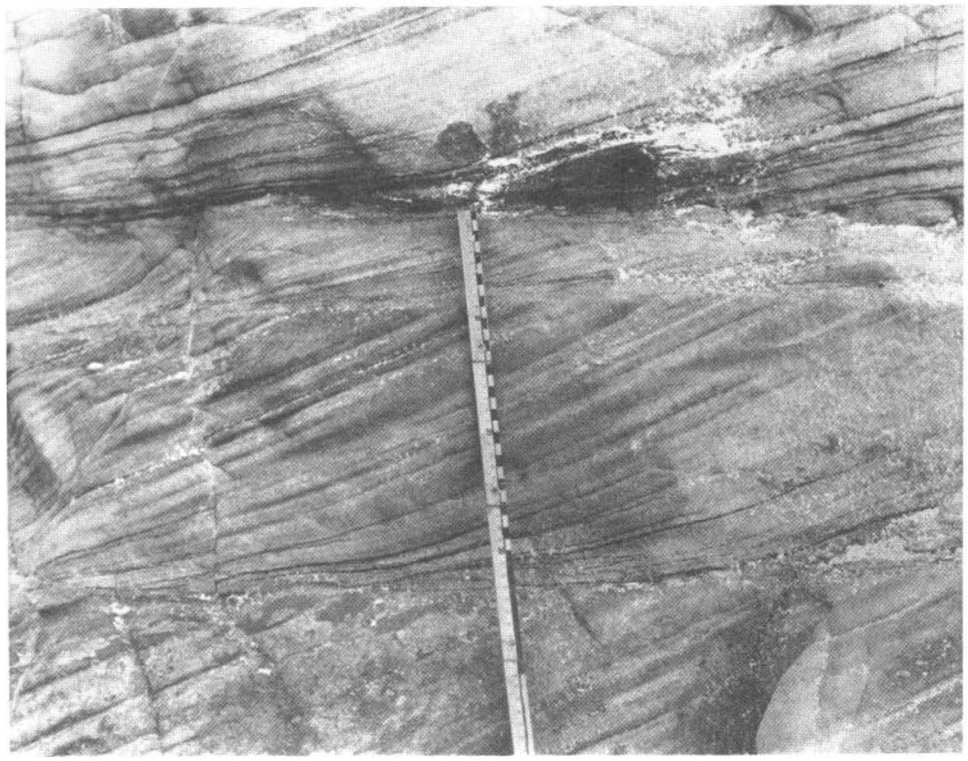


Figure 1
A crossbedded unit, near St. Monance, Fife, Scotland.

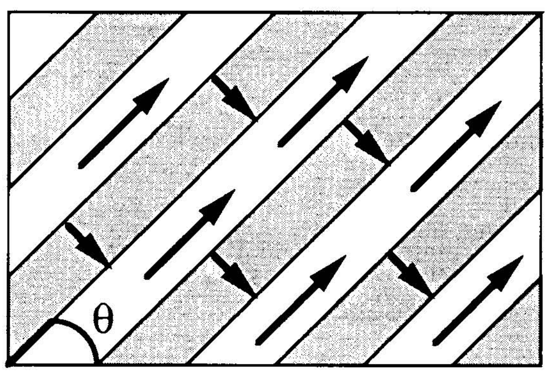


Figure 2
Flow through layers at an angle. (Low permeability laminae are represented by shading.)

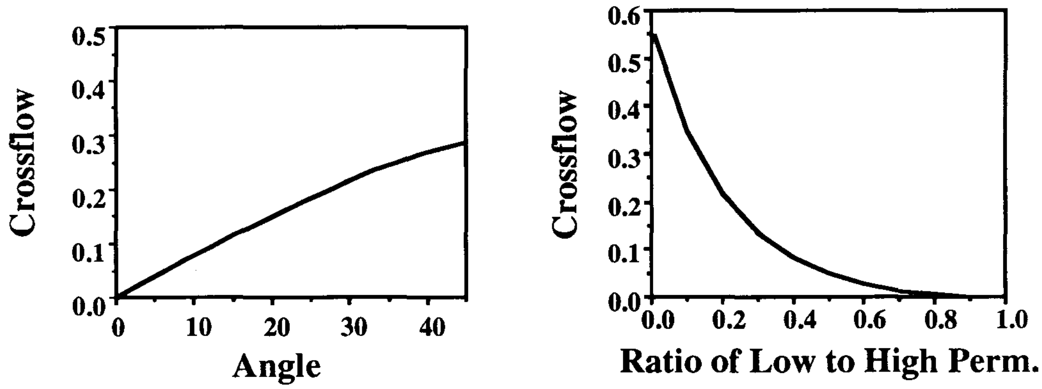


Figure 3
The dependence of 2D crossflow (measured by k_{xy}/k_{xx}) on:
a) lamina angle and b) permeability contrast.

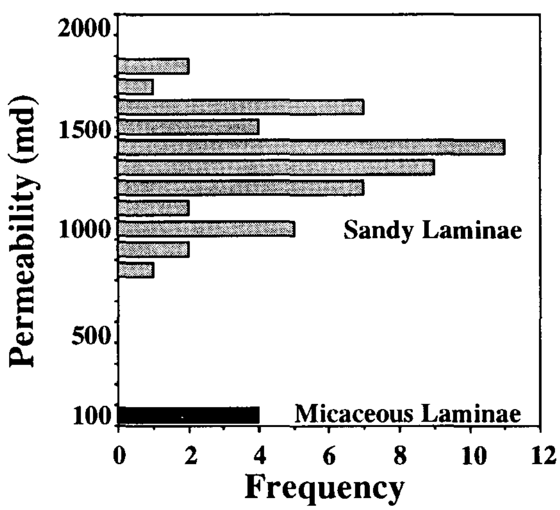


Figure 4
Histogram of probe permeameter measurements of a crossbedded sandstone sample from the Ardross Outcrop, Fife, Scotland.

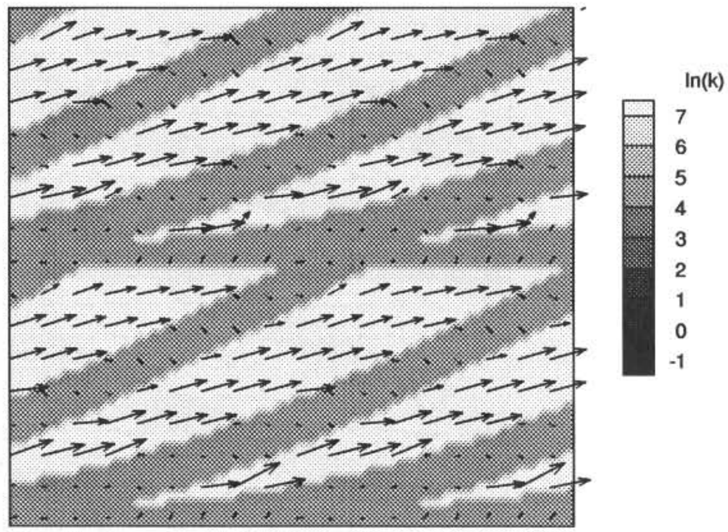


Figure 5
Flow through crossbed model 1.

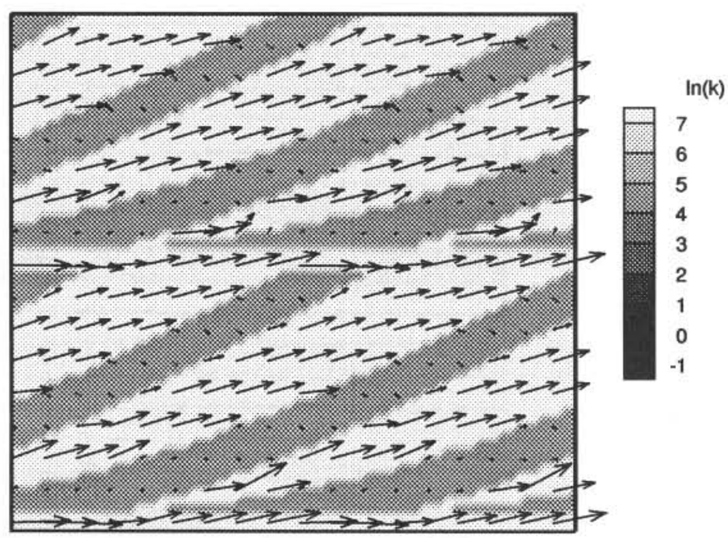


Figure 6
Flow through crossbed model 2.

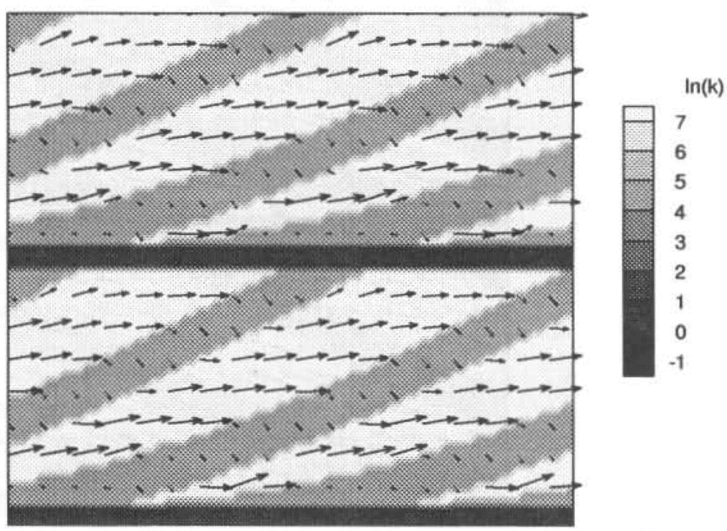


Figure 7
Flow through crossbed model 3.

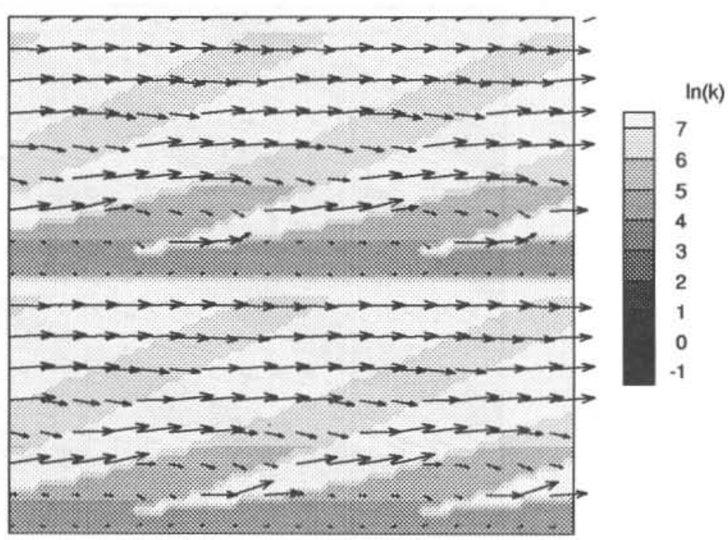


Figure 8
Flow through crossbed model 4.

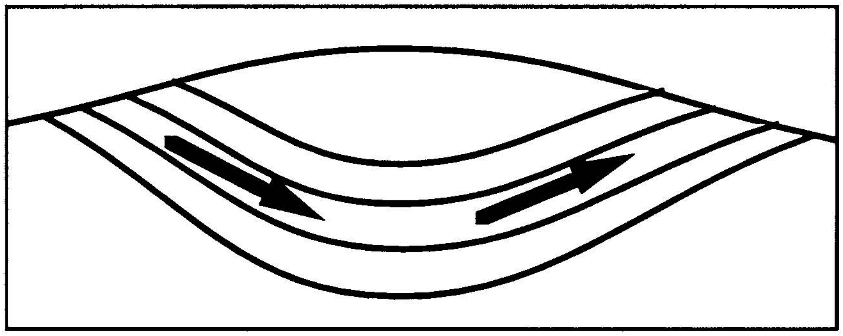


Figure 9
Flow through a model of a hummocky bed.

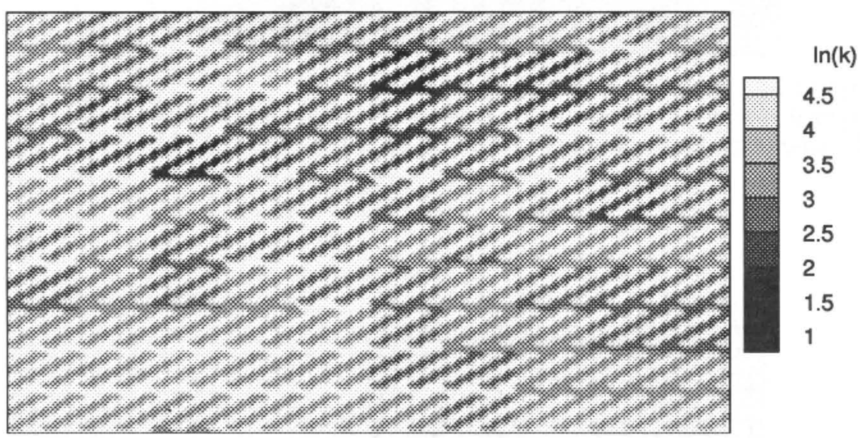


Figure 10
Stochastic ripple model.

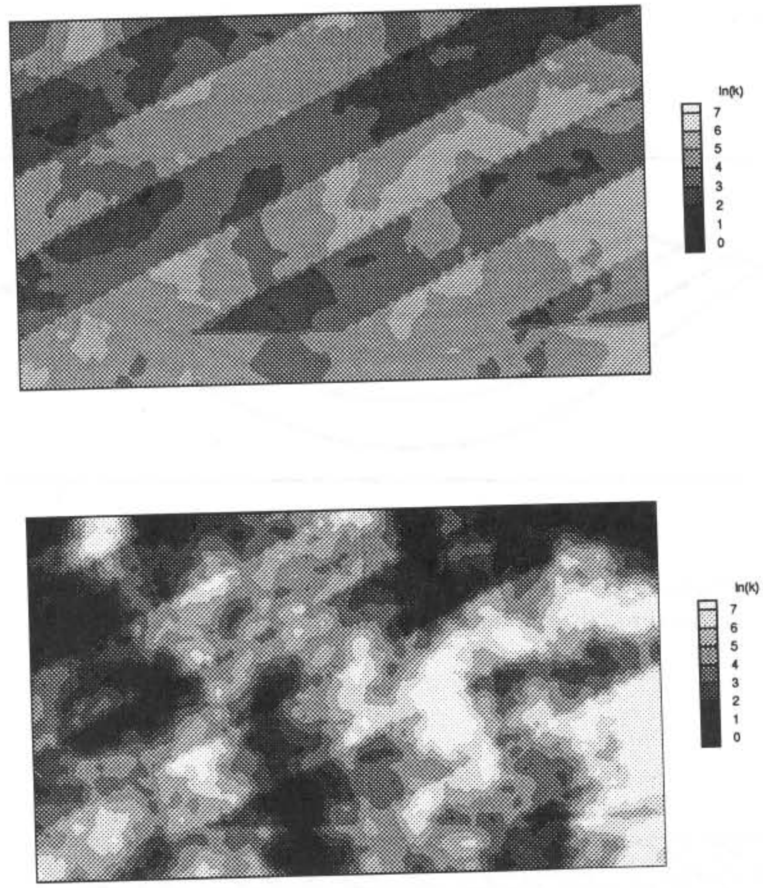
a)
b)


Figure 11
Hybrid stochastic/deterministic models. The standard deviation of ln(k) in the stochastic component is a) 0.5 and b) 2.0.

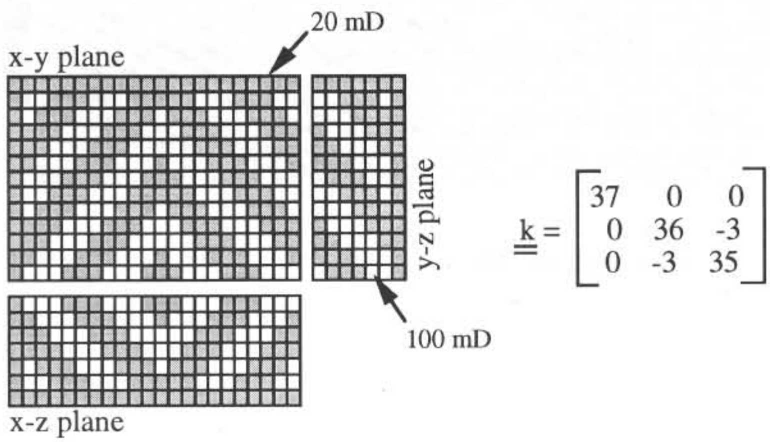


Figure 12
A 3D trough crossbed model, showing the effective permeability tensor.